Monitoria Estatística Bayesiana 1

28/03/2022

Referente ao exercício 1.17

EXAMPLE 10. Let $\mathcal{X} = \{1,2,3\}$ and $\Theta = \{0,1\}$, and consider experiments E_1 and E_2 which consist of observing X_1 and X_2 with the above $\mathcal{X}$ and the same θ, but with probability densities as follows:

	x_1 = 1	2	3
$f_0^1(x_1)$	.90	.05	.05
$f_1^1(x_1)$	.09	.055	.855

,

	x_2 = 1	2	3
$f_0^2(x_2)$	.26	.73	.01
$f_1^2(x_2)$	.026	.803	.171

.

If, now, $x_1 = 1$ is observed, the LP states that the information about θ should depend on the experiment only through $(f_0^1(1), f_1^1(1)) = (.9, .09)$. Furthermore, since this is proportional to $(.26, .026) = (f_0^2(1), f_1^2(1))$, it should be true that $x_2 = 1$ provides the same information about θ as does $x_1 = 1$. Another way of stating the LP for testing simple hypotheses, as here, is that the experimental information about θ is contained in the likelihood ratio for

the observed x. Note that the likelihood ratios for the two experiments are also the same when 2 is observed, and also when 3 is observed. Hence, no matter which experiment is performed, the *same* conclusion about θ should be reached for the given observation. This example clearly indicates the startling nature of the LP. Experiments E_1 and E_2 are very different from a frequentist perspective. For instance, the test which accepts $\theta = 0$ when the observation is 1 and decides $\theta = 1$ otherwise is a most powerful test with error probabilities (of Type I and Type II, respectively) .10 and .09 for E_1, and .74 and .026 for E_2. Thus the classical frequentist would report drastically different information from the two experiments. (And the conditional frequentist is also likely to report E_1 and E_2 differently; indeed, for E_2 it is hard to perform any sensible conditional frequentist analysis because of the three point $\mathcal{X}$ and the widely differing error probabilities.)

Referente ao exercício 1.26

If we keep in mind the inversion aspect of Statistics presented in Section 1.2, it is tempting to consider the likelihood as a generalized density in θ, whose mode would then be the maximum likelihood estimator, and to work with this density as with a regular distribution. This approach seems to have been advocated by Laplace when he suggested using the uniform prior distribution when no information was available on θ (see Examples 1.2.3–1.2.5). Similarly, Fisher introduced the fiducial approach (see Note 1.8.1) to try to circumvent the determination of a prior distribution while putting into practice the Likelihood Principle, the choice of his distribution being objective (since depending only on the distribution of the observations). However, this approach is at its most defensible when θ is a location parameter (see also Example 1.5.1), since it leads in general to paradoxes and contradictions, the most immediate being that $\ell(\theta|x)$ is not necessarily integrable as a function of θ (Exercise 1.26). The derivation of objective posterior distributions actually calls for a more advanced theory of *non-informative* distributions (see Chapter 3), which shows that the likelihood function cannot always be considered the most natural posterior distribution.

Fazer 1.25

Referente ao exercício 1.37

Fazer 1.38

Referente ao exercício 1.41

https://rss.onlinelibrary.wiley.com/doi/epdf/10.1111/j.2517-6161.1993.tb01477.x