12/03/2021

Aplicações Diferenciáveis: $F: U \to \mathbb{R}^n$, $U \subseteq \mathbb{R}^m$ aberto!

$f = (f_1, \dots, f_n)$, $f_i: U \to \mathbb{R}$

Diferenciabilidade: F é diferenciável em $a \in U$, existe uma trans. linear $T: \mathbb{R}^m \to \mathbb{R}^n$,

$$F(\underbrace{a+v}_{\in U}) - F(a) = T \cdot v + \underbrace{r(v)}_{\text{resto}}$$

$$\lim_{v \to 0} \frac{r(v)}{\|v\|} = 0$$

Quando $m = n = 1$,

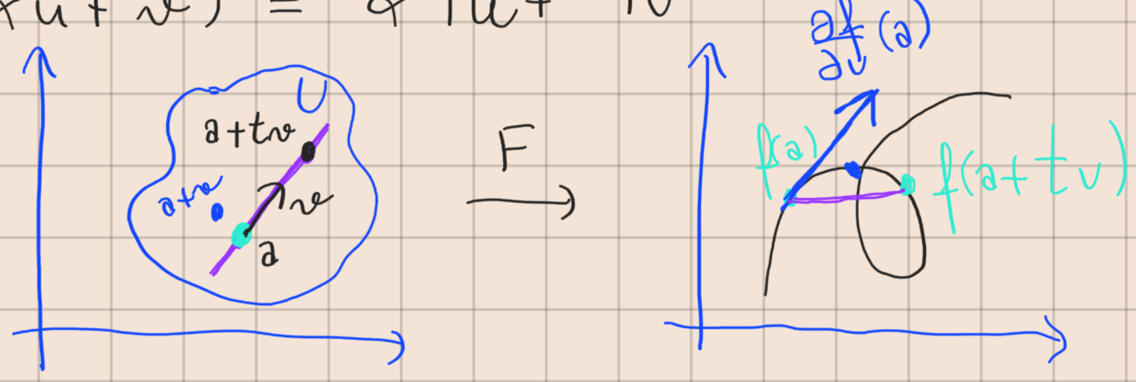
$$f(a+v) - f(a) = T \cdot v + r(v)$$

$$\Rightarrow r(v) = f(a+v) - f(a) - T \cdot v$$

$$\Rightarrow \lim_{v \to 0} \frac{r(v)}{|v|} = \lim_{v \to 0} \frac{f(a+v) - f(a)}{|v|} - \frac{T \cdot v}{|v|} = 0$$

$$\Rightarrow \lim_{v \to 0} \frac{f(a+v) - f(a)}{|v|} = T = f'(a)$$

$$T(\alpha u + v) = \alpha T u + T v$$



$$F(x,y) = (x^2y,\ xy^2)$$

$$J_F(x,y) = \begin{pmatrix} 2xy & x^2 \\ y^2 & 2xy \end{pmatrix}$$

$$\lim \frac{f(a+h) - f(a)}{h} = f'(a)$$

$$\lim \frac{f(a+h) - f(a) - f'(a).h}{h} = 0$$

$$r(h) = f(a+h) - f(a) - f'(a).h$$

$o(h)$

$U \subseteq \mathbb{R}$ convexo

* $\forall a,b \in U, \quad t \in [0,1], \quad at + b(1-t) \in U$

$$\alpha : U \to \mathbb{R}^n$$
$$t \mapsto (\alpha_1(t), \dots, \alpha_n(t))$$

$$s \in U,\ J_\alpha(s) = \begin{bmatrix} \frac{d}{dt}\alpha_1(s) \\ \vdots \\ \frac{d}{dt}\alpha_n(s) \end{bmatrix}$$