Monitoria 12/08

- $y' = F(y(t)) \to$ Proposta
- $y_t = F(y_x(x,t))$

Preços de ativos

$$y(x,t) \qquad \forall (x,t) \in U \times [0,T]$$

Método da Características: Resolvendo uma EDP

$$F\left(\frac{\partial^k u}{\partial x_1^k}, \dots, \frac{\partial^k u}{\partial x_n^k}, \dots, \frac{\partial u}{\partial x_1}, \dots, \frac{\partial u}{\partial x_n}, u, x_1, \dots, x_n\right) = 0$$

O que o método resolve:

$$F(\nabla u, u, x) = 0$$

$\nabla \to$ Nabla

$x = (x_1, \dots, x_n)$

$\nabla u = (u_{x_1}, \dots, u_{x_n})$

$u(x) = g(x), \quad x \in \Gamma \to$ condição de fronteira

$u(x, t=0) = g(x) \to$ Condição inicial

Equação do Transporte:

Notação:

$$Du = (u_{x_1}, \dots, u_{x_n}) = \nabla u = D_x u$$
$$[D^2 u]_{ij} = u_{x_i x_j} = D^2_x u$$

constante

$$u_t(x,t) + b \cdot D_x u(x,t) = 0, \quad \forall (x,t) \in \mathbb{R}^n \times (0, +\infty)$$

$$u_t + b u_x = 0 \rightarrow c = (1, b)$$

$$u_t + b \cdot (u_x, u_y) = u_t + b_1 u_x + b_2 u_y = 0 \rightarrow c = (1, b_1, b_2)$$

Defino o vetor $c = (1, b) \in \mathbb{R}^{n+1}$

$$c \cdot (u_t, u_{x_1}, \dots, u_{x_n}) = 0$$

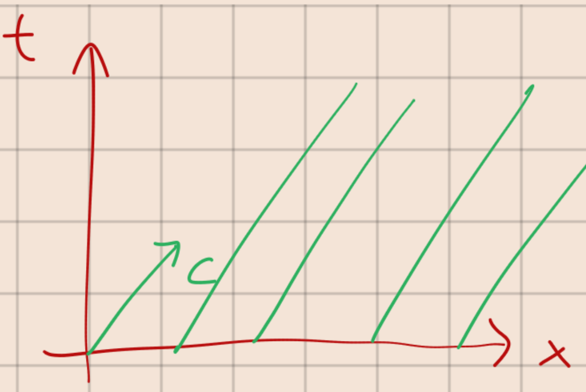
$$= c \cdot \nabla u(x,t) = 0, \quad \forall (x,t)$$

Derivada direcional:

$$\frac{\partial f}{\partial v} = \lim_{h \to 0} \frac{f(x + v \cdot h) - f(x)}{h}$$
$$= \nabla f \cdot v$$

$$\frac{\partial u}{\partial c} = c \cdot \nabla u = 0 \Rightarrow u(x,t) \equiv \text{constante}$$

na direção c.



$$z(s) = u(t+s,\ x+bs)$$
$$= u((t,x) + s(1,b))$$
$$= u((t,x) + s\cdot c)$$

$$\dot{z}(s) = u_t \cdot 1 + (u_{x_1}, \dots, u_{x_n}) \cdot b = 0$$

$$u(0,x) = f(x)$$

$$z(-t) = u(t-t,\ x + b\cdot(-t))$$
$$= u(0,\ x-bt) = f(x-bt)$$

$$\Rightarrow z(s) \equiv f(x-bt)$$

$$u(x,t) = f(x-bt)$$

- Defina as curvas $\gamma(s) = (a_1(s), \dots, a_n(s)) \in \mathbb{R}^n$

$$F(Du, u, x) = 0$$
$$z(s) = u(\gamma(s))$$
$$p(s) = D_x u(\gamma(s)) = (u_{x_1}(\gamma(s)), u_{x_2}(\gamma(s)), \dots, u_{x_n}(\gamma(s)))$$
$$p^i(s) = u_{x_i}(\gamma(s))$$

- $$\frac{d}{ds} p^i(s) = \frac{d}{ds} u_{x_i}(\gamma(s)) = \sum_{j=1}^{n} \boxed{u_{x_i x_j}} \boxed{\frac{d}{ds} \gamma_j(s)}$$

- $$F(Du, u, x) = 0$$

$$F(\underbrace{Du(\gamma(s))}_{= p(s)}, \underbrace{u(\gamma(s))}_{= z(s)}, \gamma(s)) = 0$$

$$F(p(s), z(s), \gamma(s)) = 0$$

$$\frac{\partial}{\partial x_i} F(Du, u, x) = \sum_{j=1}^{n} F_{u_{x_j}} \cdot u_{x_j x_i} + F_u \cdot u_{x_i} + F_{x_j} = 0$$

$$= \sum_{j=1}^{n} \boxed{F_{p_j}} \boxed{u_{x_j x_i}} + F_z \cdot u_{x_i} + F_{x_j} \frac{dx_j}{dx_i} = 0$$

Posso fazer

$$= \frac{d}{ds} p^i(s) + F_z \cdot u_{x_i} + F_{x_i} = 0$$

Digo que

$$\frac{d}{ds} \gamma_j(s) = F_{p_j}(p(s), z(s), \gamma(s))$$

$$\frac{d}{ds} p^i(s) = -F_z \cdot u_{x_i} - F_{x_i} = -F_z \cdot p^i - F_{x_i}$$

$$\frac{d}{ds} z(s) = \sum_{j=1}^{n} u_{x_j} \cdot \frac{d}{ds} \gamma^j(s)$$
$$= \sum_{j=1}^{n} u_{x_j} \cdot F_{p_j}$$

$$\begin{cases} \dot{p}(s) = -D_2 F \cdot p - D_x \\ \dot{z}(s) = D_p F \cdot p \\ \dot{\gamma}(s) = D_p F \end{cases}$$

→ Sistema de $2n+1$ EDOs

↳ curvas projetas

$$\begin{cases} u_t + b(x,y) \cdot (u_x, u_y) = 0 \\ u(x,0) = g(x), \quad x > 0 \end{cases}$$

$$b(x,y) = (-y, x)$$

$$\begin{cases} \overset{p_1}{\boxed{u_t}} - y \overset{p_2}{\boxed{u_x}} + x \overset{p_3}{\boxed{u_y}} = 0 \\ u(x,0) = g(x), \quad x > 0 \end{cases}$$

$$\dot{\gamma}(s) = (1, -y, x)$$
$$\dot{z}(s) = (1, -y, x) \cdot (u_t, u_x, u_y)$$
$$=$$