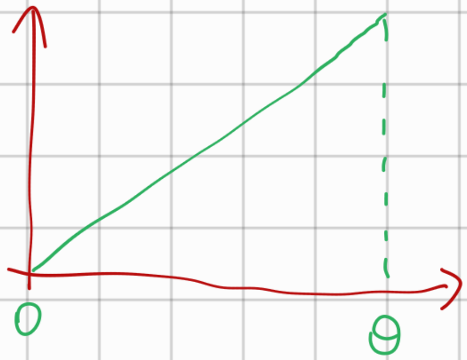
Monitoria 13/07

16 (KN): $X_1, \dots, X_n \sim P_\theta$

$$f_\theta(x) = \begin{cases} \frac{2x}{\theta^2}, & x \in (0, \theta) \\ 0, & c.c. \end{cases}$$



a) $T(x) = \max(x_1, \dots, x_n)$

b) CDF (Função de distribuição acumulada)

$$F_\theta(t) = \mathbb{P}_\theta(T(x) \leq t)$$
$$= \mathbb{P}_\theta(\max(x_1, \dots, x_n) \leq t)$$
$$\Updownarrow$$
$$x_1 \leq t,\ x_2 \leq t, \dots, x_n \leq t$$
$$= \mathbb{P}_\theta(X_1 \leq t, X_2 \leq t, \dots, X_n \leq t)$$
$$= \prod_{i=1}^{n} \mathbb{P}_\theta(X_i \leq t)$$
$$= \mathbb{P}_\theta(X_1 \leq t)^n$$

$$\mathbb{P}_\theta(X_1 \leq t) = \int_0^t \frac{2x}{\theta^2}\, dx = \left.\frac{x^2}{\theta^2}\right|_0^t = \frac{t^2}{\theta^2},$$

se $t \leq \theta$. Se $t > \theta \Rightarrow \mathbb{P}_\theta(X_1 \leq t) = 1$

$$F_\theta(t) = \begin{cases} \left(\frac{t^2}{\theta^2}\right)^n, & t \in [0, \theta], \\ 0, & t < 0 \\ 1, & t > \theta \end{cases}$$

$$f_{T,\theta}(t) = \frac{d}{dt} F_\theta(t)$$

$$= \frac{2n\, t^{2n-1}}{\theta^{2n}} \mathbb{1}(t \in (0, \theta))$$

Extra:

$$P(X_1 = y_1, \dots, X_n = y_n \mid X_{(1)} = x_1, \dots, X_{(n)} = x_n) = \frac{1}{n!}$$

$$T(x) = \boxed{(2, 4, 7, 8, 10)}$$

$$\left\{\begin{matrix} X_1 = 2 & n = 5 \\ X_2 = 10 & (n-1) = 4 \\ X_3 = 7 & \vdots \\ X_4 = 4 & \vdots \\ X_5 = 8 & 1 \end{matrix}\right\} \quad n! = 5!$$

Independência $\Rightarrow$ Permutáveis

$$f_\theta(x_1, x_2) = f_\theta(x_2, x_1)$$

c) $T(x) = \max(x_1, \dots, x_n)$ é completa.

= Uma estatística T é completa se

mensurável (contínua)

$E[g(T)] = 0 \Rightarrow g(t) = 0$ (quase sempre)

$\forall \theta \in \Theta$

Seja g uma função

$$E_\theta[g(T)] = \int_0^\theta g(t) \cdot \frac{t^{2n-1}}{\theta^{2n}} \cdot 2n \, dt$$

$$= \cancel{\frac{2n}{\theta^{2n}}} \boxed{\int_0^\theta g(t)\, t^{2n-1}\, dt} = 0$$

$$\int_0^\theta g(t)\, t^{2n-1}\, dt = 0$$

TFC $\Rightarrow$ $g(\theta)\,\theta^{2n-1} = 0, \quad \forall \theta > 0$

$\Rightarrow g(\theta) = 0, \quad \forall \theta > 0$

$\Rightarrow$ T é completa

Regra de Leibniz

$$\frac{d}{dx}\int_a^b f(t,x)\, dt = \int_a^b \frac{\partial}{\partial x} f(t,x)\, dt$$

$(2, 4, 10, 7, 8)$ quanto $(4, 10, 2, 7, 8)$

Não consigo diferenciar. $(n!)$

$P(X_1 = x_1, X_2 = x_2 \mid X_{(1)} = x_{(1)}, X_{(2)} = x_{(2)})$

Se $\underline{P_\theta}$ é absolutamente contínuo,

$$f_\theta(x_1, \dots, x_n) = \prod_{i=1}^{n} f_\theta(x_i)$$

$$= \prod_{i=1}^{n} f_\theta(x_{(i)}), \quad x_{(1)} < \dots < x_{(n)}$$

$$= \boxed{g(T(x) \mid \theta)}$$